Extending the GPS Paradigm to Space Exploration

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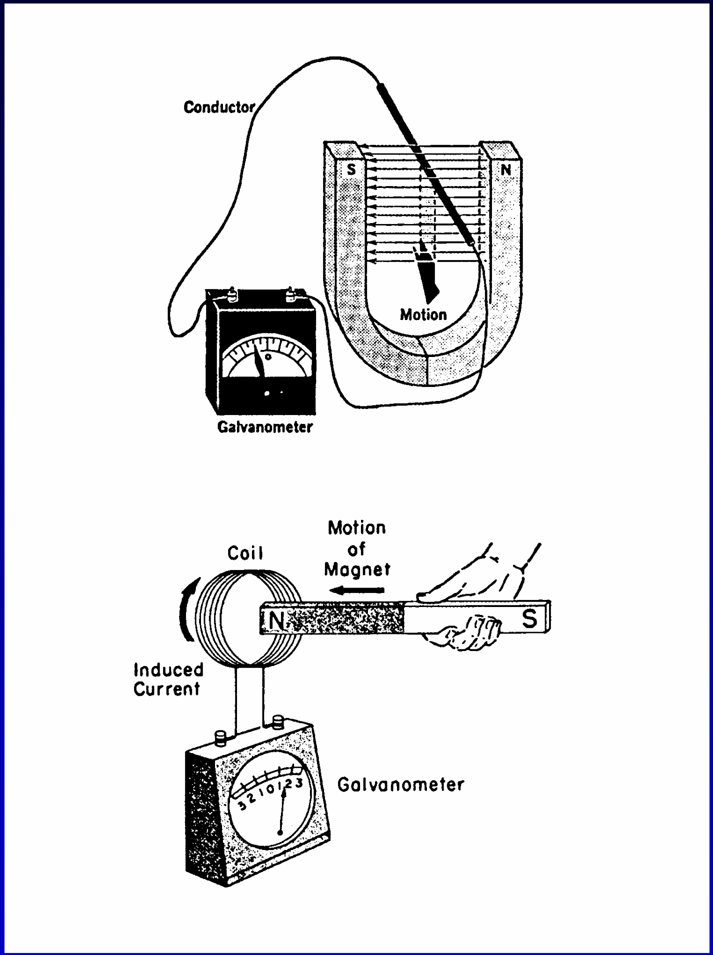
Einstein's paper on special relativity (1905)



Einstein postulated that the laws of electrodynamics (Maxwell's equations) should hold in every inertial frame of reference

Maxwell's equations predict the existence of electromagnetic waves that propagate at the unique speed *c* (speed of light) depending only on fixed electrical constants μ_0 and ε_0 ,

$$c = 1/\sqrt{\mu_0 \, \varepsilon_0}$$

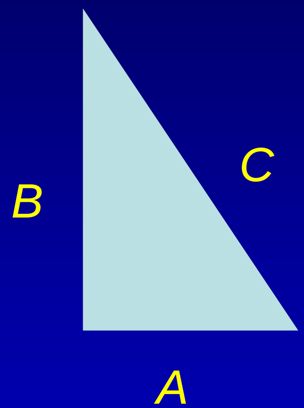
Speed of light *c* must be the same in every inertial frame

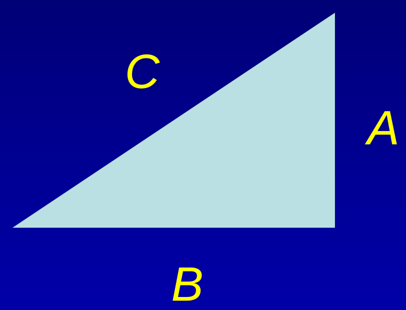
Current depends only on the relative motion of conductor and magnet. It does not depend on whether conductor or magnet is in motion.

Invariant space-time interval

For light signals,
$$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2=c^2$$

$$ds^2=dx^2+dy^2+dz^2-c^2dt^2=0$$





Pythagorean theorem $A^2+B^2=C^2$ (invariant with respect to orientation)

Space-time interval $ds^2=dx^2+dy^2+dz^2-c^2dt^2=$ invariant

For clocks, $ds^2=-c^2d\tau^2=dx^2+dy^2+dz^2-c^2dt^2$

Relativistic effects on a transported clock

Three effects contribute to the net relativistic effect on a transported clock

- **Velocity (time dilation)**
  - Makes transported clock run slow relative to a clock on the geoid
  - Function of speed only
- **Gravitational potential (redshift)**
  - Makes transported clock run fast relative to a clock on the geoid
  - Function of altitude only
- **Sagnac effect (rotating frame of reference)**
  - Makes transported clock run fast or slow relative to a clock on the geoid
  - Depends on direction and path traveled

Around the world atomic clock experiment

J.C. Hafele and R.E. Keating (1971)

Around the world atomic clock experiment

(Flying clock – Reference clock)

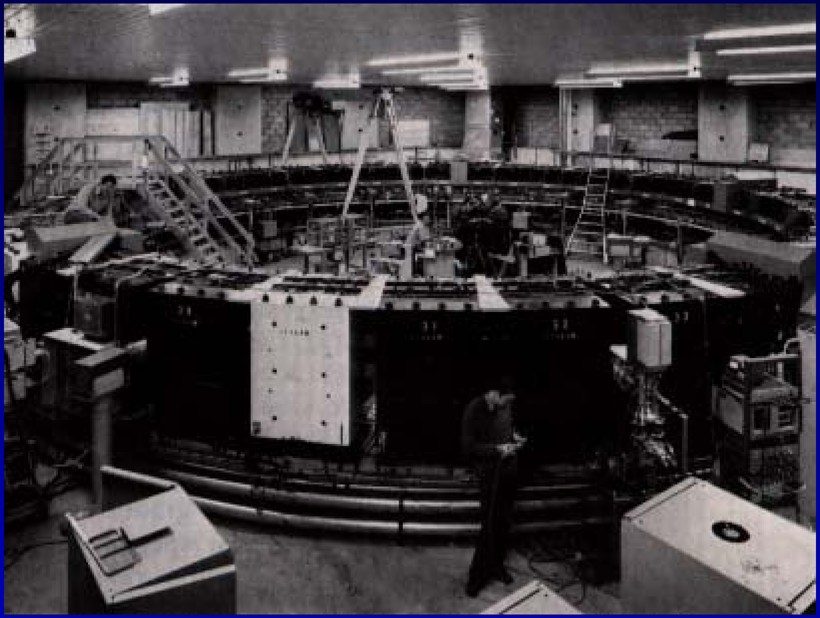
$$v_2 = v' + \omega R \qquad\qquad v_1 = \omega R$$

$$\Delta\tau_2 - \Delta\tau_1 \approx \left[-\frac{1}{2c^2}(v_2{}^2 - v_1{}^2) + \frac{g\,h}{c^2} \right]\Delta\tau_1 = \left[-\frac{1}{2c^2}\left(v'^2 + 2\,v'\omega\,R\right) + \frac{g\,h}{c^2} \right]\Delta\tau_1 = -\frac{2\pi R}{c^2}\left(\frac{1}{2}|v'| \pm \omega\,R\right) + \frac{g\,h}{c^2}\Delta\tau_1$$

predicted effect	*direction*	
	East	West
Velocity (time dilation)	– 51 ns	– 47 ns
Sagnac effect	– 133 ns	+ 143 ns
Gravitational potential (redshift)	+ 144 ns	+ 179 ns
Total	– 40 ± 23 ns	+ 275 ± 21 ns
Measured	– 59 ± 10 ns	+ 273 ± 7 ns

CERN muon experiment

J. Bailey, *et al.* (1968, 1977)



CERN muon storage ring

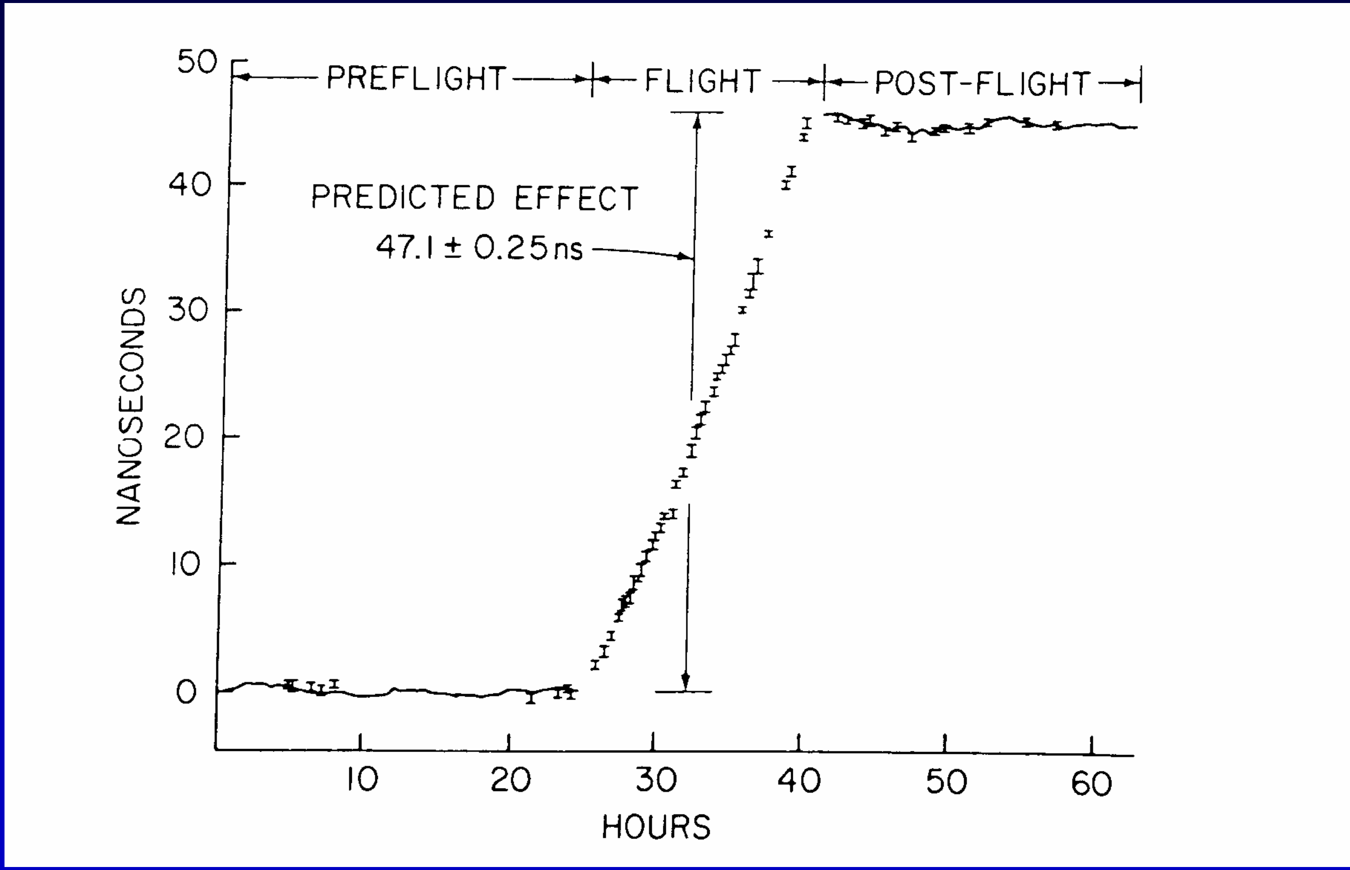
$r = 7\ \mathrm{m}$ $\qquad p = 3.094\ \mathrm{GeV}/c$ $\qquad v/c = 0.9994$

$$\gamma = (1 - v^2/c^2)^{-1/2} = [1 - (0.9994)^2]^{-1/2} = 29.3$$

$$\Delta\tau_{\mathrm{lab}} = \gamma\,\Delta\tau_{\mathrm{muon}}$$

Gravitational redshift of an atomic clock

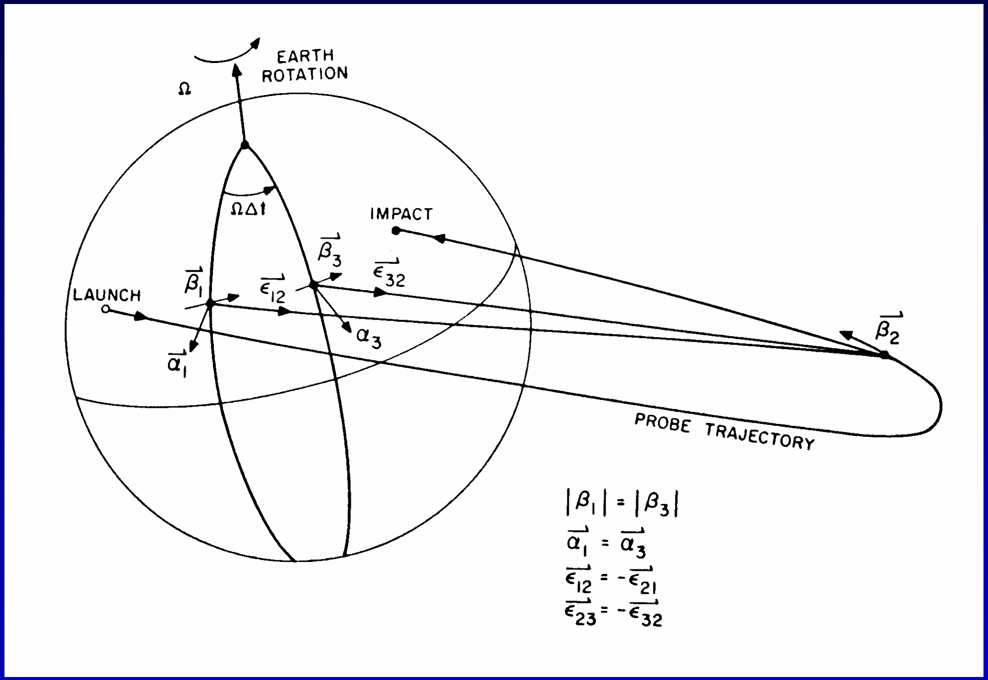
C.O. Alley, *et al.* (1975)



Gravitational redshift	52.8 ns
Time dilation	5.7 ns
Net effect	47.1 ns

Gravitational redshift

R.F.C. Vessot *et al.* (1976)



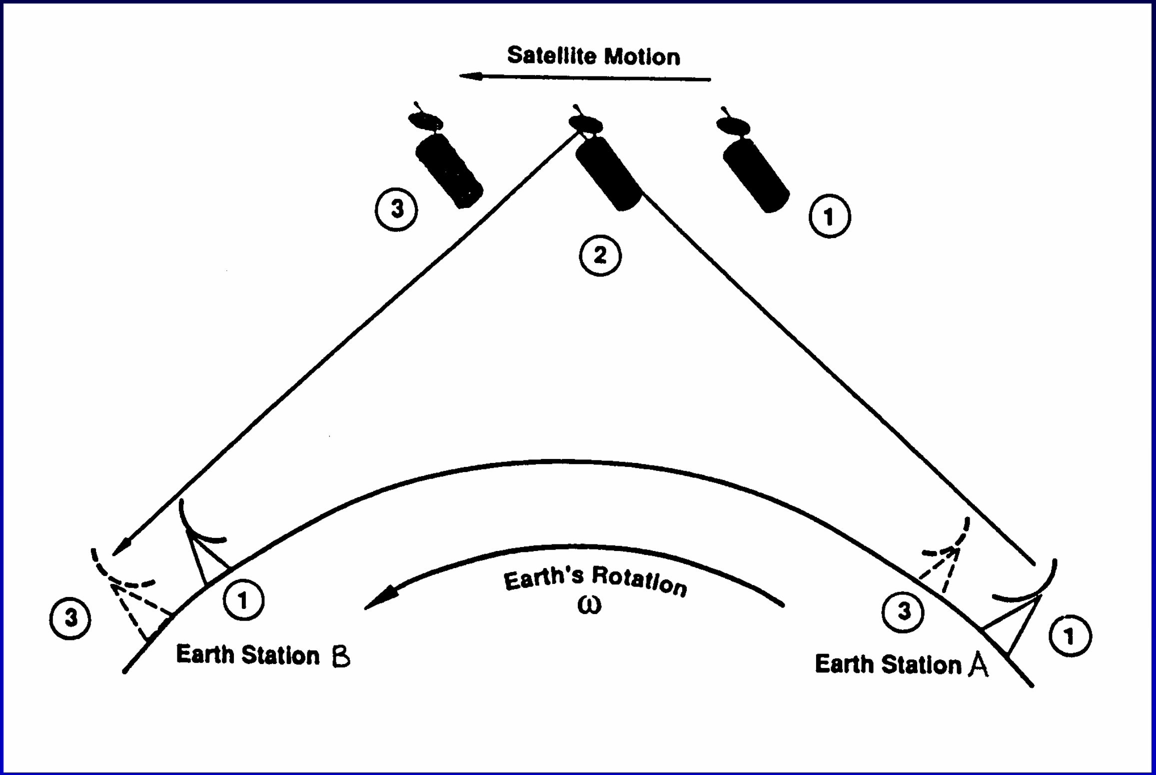
Gravity Probe A

At the 10,000 km altitude apogee,

$$\frac{\Delta f}{f} \approx -\frac{GM}{c^2}\left(\frac{1}{r}-\frac{1}{R}\right) = -\frac{398\,600.5\ \mathrm{km}^3/\mathrm{s}^2}{(3.00\times10^5\ \mathrm{km/s})^2}\left(\frac{1}{16\,378\ \mathrm{km}}-\frac{1}{6378\ \mathrm{km}}\right) = 4.2\times10^{-10}$$

Sagnac effect (TWSTT)

NIST to USNO via *Telstar 5* at 97° WL



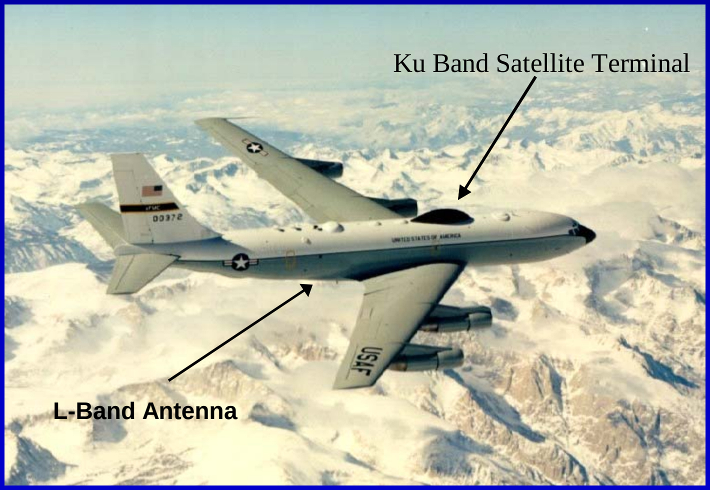
Uplink	24.1 ns
Downlink	57.7 ns
Total Sagnac correction	81.1 ns

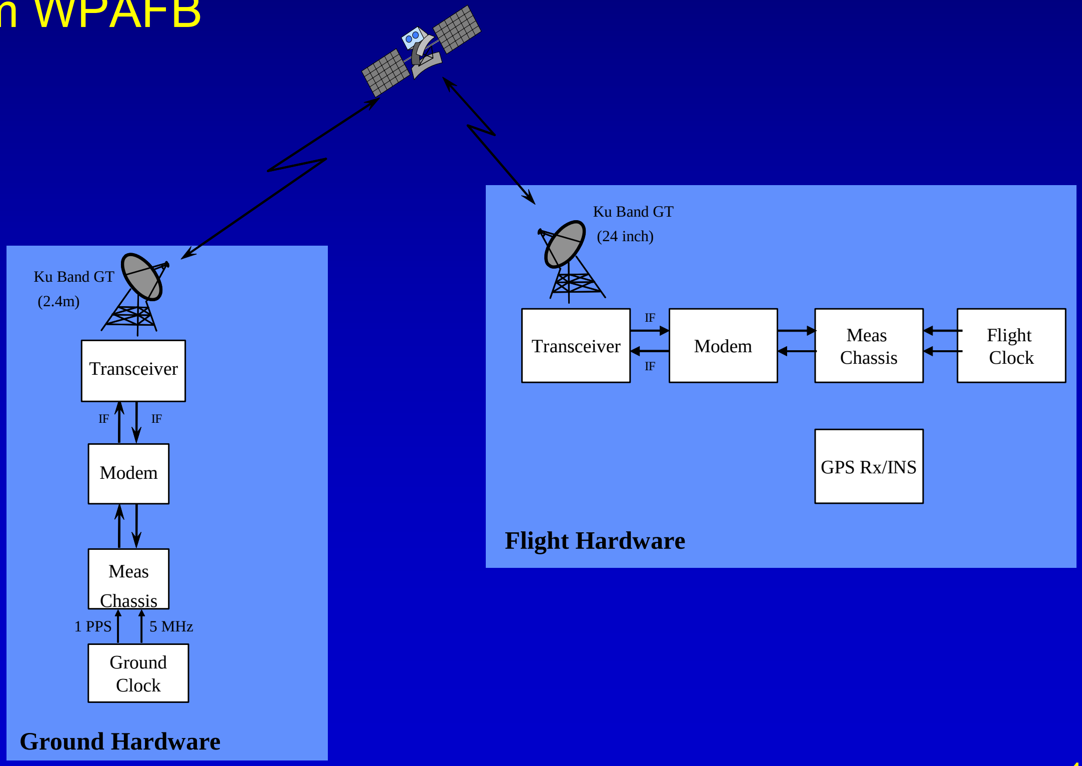
TWTT Flight Tests

Tests conducted by Timing Solutions Corp., Zeta Associates, and AFRL

Flight clock data collected on a C-135E aircraft to demonstrate TWTT in background of an active communications channel

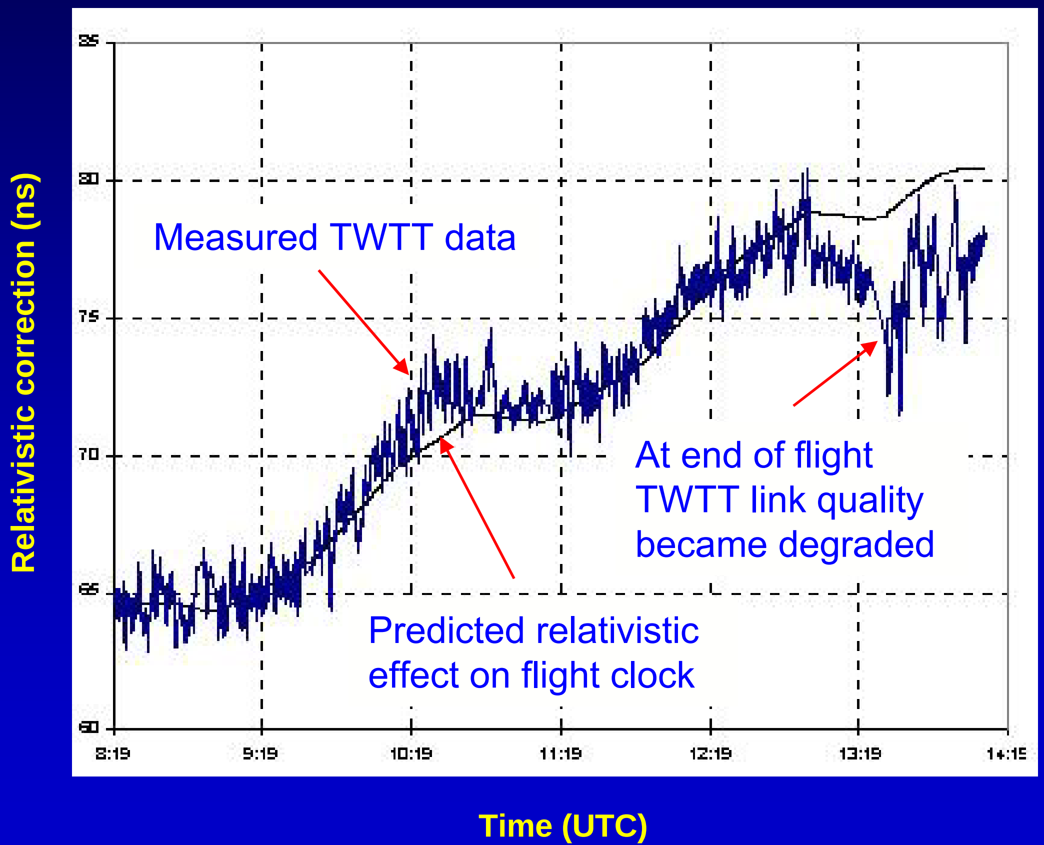
6 flights in November 2002 from WPAFB





Prediction of Relativistic Effects

Comparison of Measured Data with Prediction (Flight Clock – Reference Clock)



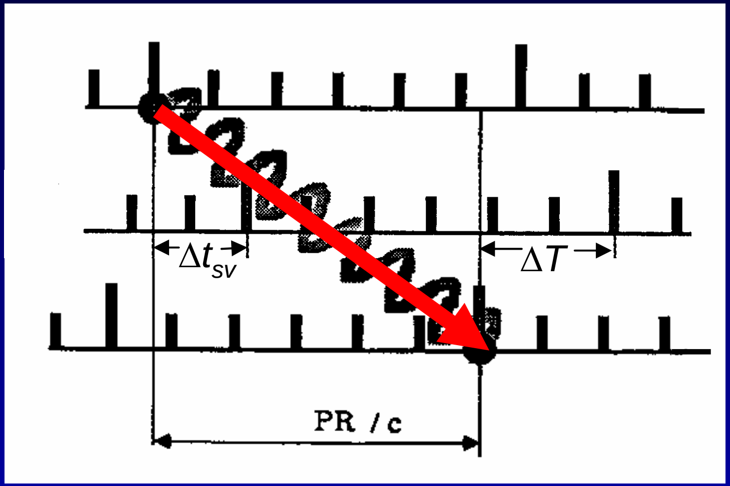
Relativistic Corrections

Velocity (time dilation) $$\Delta\tau = -\frac{1}{2\,c^2}\sum_{i=1}^{N} v_i^{\,2}\,\Delta t_i$$

Gravitation (redshift) $$\Delta\tau = \frac{g}{c^2}\sum_{i=1}^{N}(h_i - h_0)\,\Delta t_i$$

Sagnac effect $$\Delta\tau = -\frac{\omega}{c^2}\sum_{i=1}^{N} R_i^{\,2}\cos^2\phi\;\Delta\lambda_i$$

GPS measurement is pseudorange by alignment of satellite and receiver codes



PRN sequence transmitted by satellite

GPS Time maintained by MCS

Replica PRN sequence generated in receiver

$$PR = D + c\,(\Delta T - \Delta t_{sv} + \Delta t_{iono} + \Delta t_{tropo}) \qquad \Delta t_{sv} = \Delta t_{sv}^{*} + \Delta t_{rel}$$

Satellite broadcasts its own ephemeris in navigation message.

Receiver measures propagation time of signal (pseudorange) by autocorrelation between transmitted and replica pseudorandom noise (PRN) codes.

Four pseudorange measurements plus corrections yield receiver position and time.

Relativistic effects

Satellite clock in Earth-Centered Inertial (ECI) frame of reference

$$\Delta t = \int_A^B \left\{ 1 + \underbrace{\frac{1}{2}\frac{v^2}{c^2}}_{\text{time dilation}} + \underbrace{\frac{1}{c^2}(U - W_0)}_{\text{redshift}} \right\} d\tau$$

t = coordinate time read by clocks on the geoid
τ = proper time read by satellite clock
v = satellite velocity, $0.5\, v^2 / c^2 \cong 7.2$ μs/day
U = gravitational potential, $U / c^2 \cong 14.4$ μs/day
W_0 = geopotential, $W_0 / c^2 \cong 60.2$ μs/day

Light signal in rotating Earth-Centered Earth-Fixed (ECEF) frame of reference

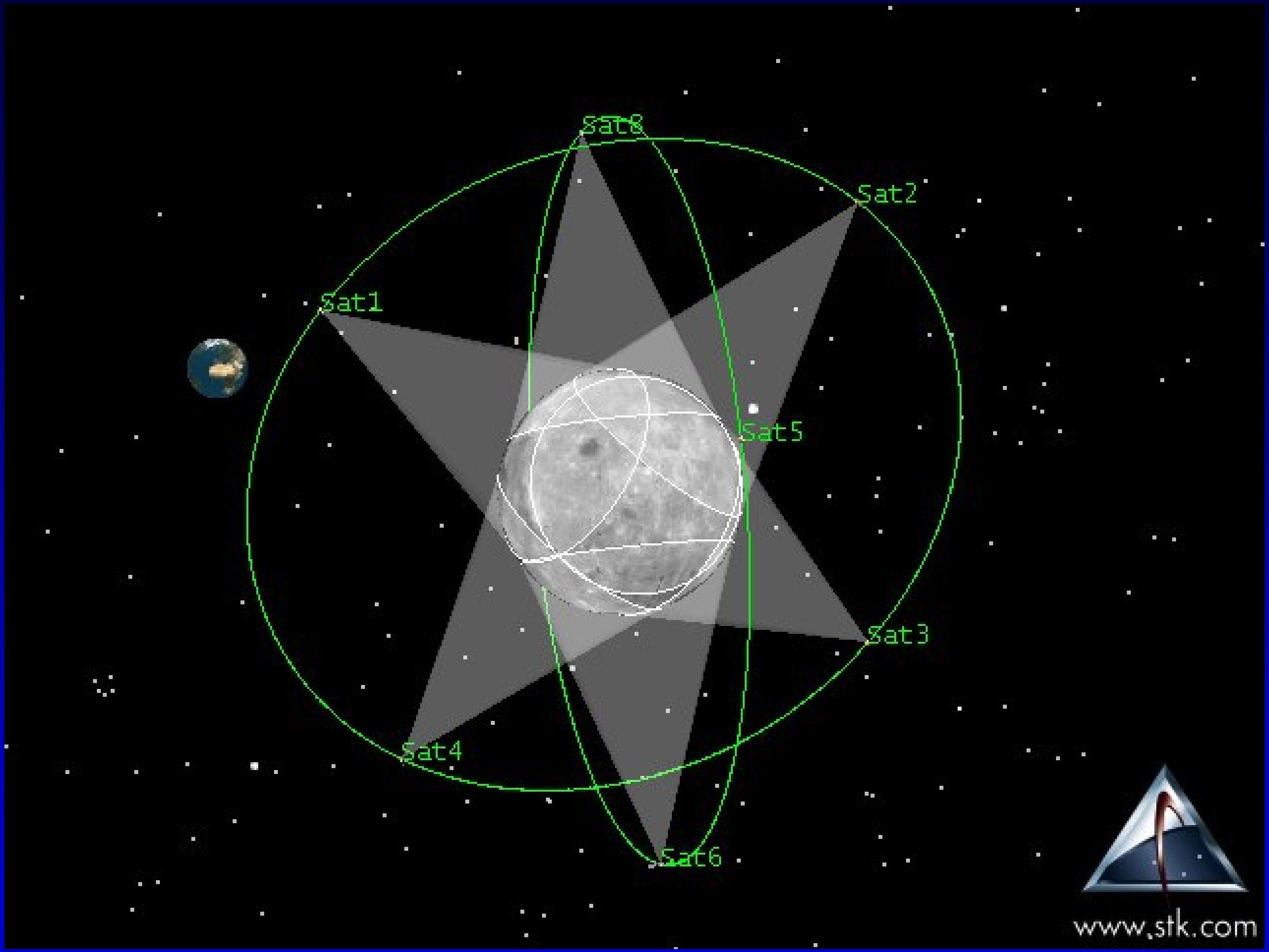
$$\Delta t = \frac{D}{c} + \underbrace{\frac{2\,\omega\, A}{c^2}}_{\text{Sagnac effect}}$$

t = coordinate time read by clocks on the geoid
D = geometric distance from satellite to receiver at coordinate time of transmission
ω = angular velocity of Earth
A = equatorial projection of triangle formed by satellite, receiver, and center of Earth

Relativistic effects incorporated in the GPS (satellite clock – geoid clock)

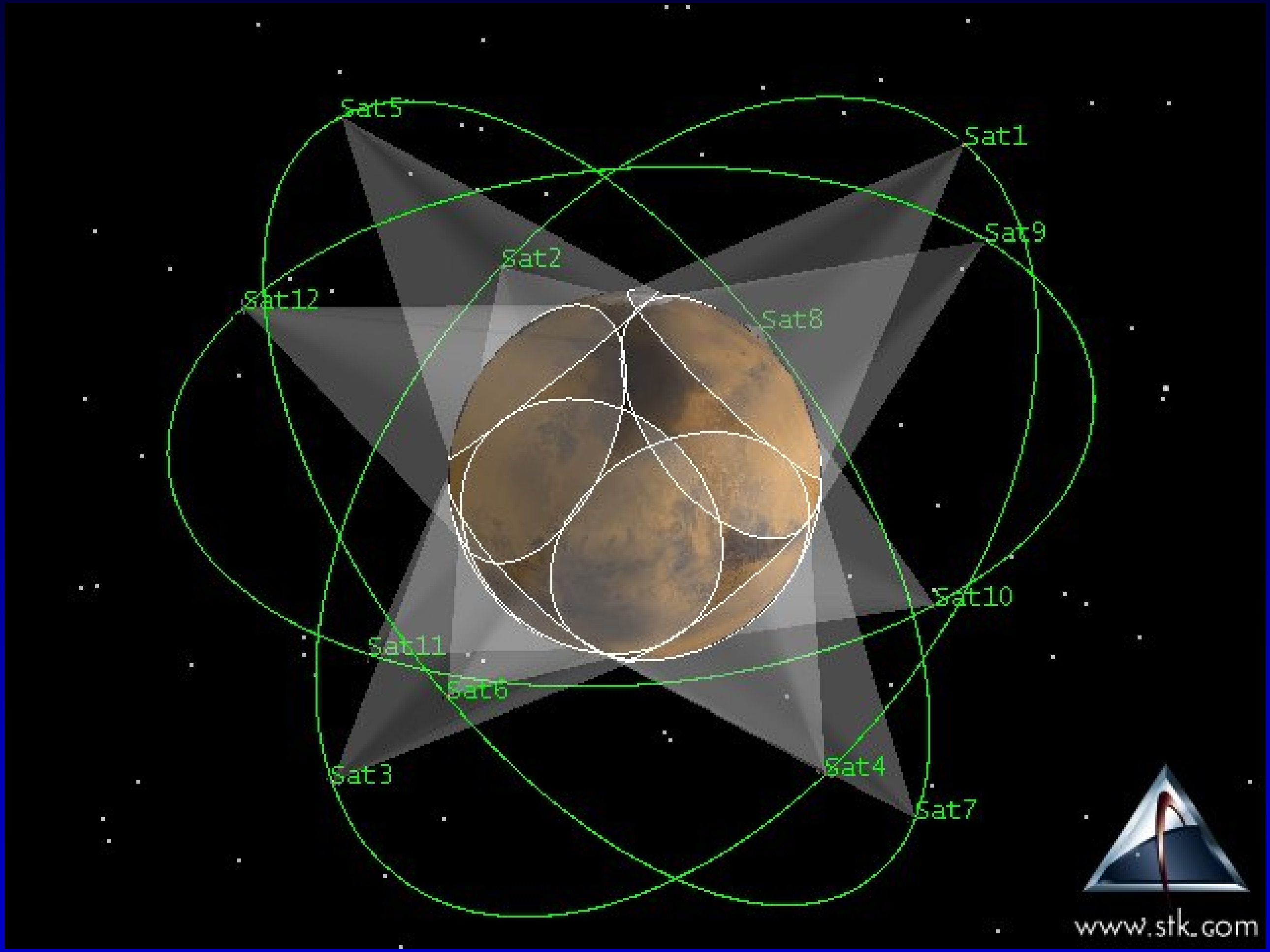
Time dilation:	− 7.2 μs per day
Gravitational redshift:	+ 45.8 μs per day
Net secular effect:	+ 38.6 μs per day
Residual periodic effect:	46 ns amplitude for e = 0.02
Sagnac effect:	133 ns maximum for receiver at rest on geoid

8 satellite polar lunar constellation



8 satellites, 2 orbital planes, 4 satellites per plane, 3 lunar radii

12 satellite Mars constellation



12 satellites, 3 orbital planes, 4 satellites per plane, 2.5 Mars radii

Relativistic corrections to a clock on Mars

- Atomic clock (e.g., rubidium) on Mars
- Potential applications of Earth-Mars synchronization
  - Very Long Baseline Interferometry (VLBI)
  - Interplanetary radionavigation references
  - Refined tests of general relativity
- Transformation between Terrestrial Time (TT) and Barycentric Coordinate Time (TCB)

$$\mathrm{TCB} - \mathrm{TT} = \frac{1}{c^2} \int \left(U_{E\,ext}(\mathbf{r}_E) + \frac{1}{2} v_E{}^2 \right) dt + L_G\, \Delta D + \frac{1}{c^2} \mathbf{v}_E \cdot (\mathbf{r} - \mathbf{r}_E)$$

- Transformation between Mars Time (MT) and Barycentric Coordinate Time (TCB)

$$\mathrm{TCB} - \mathrm{MT} = \frac{1}{c^2} \int \left(U_{M\,ext}(\mathbf{r}_M) + \frac{1}{2} v_M{}^2 \right) dt + L_M\, \Delta D + \frac{1}{c^2} \mathbf{v}_M \cdot (\mathbf{r}' - \mathbf{r}_M)$$

- Gravitational propagation time delay

Orbital semimajor axis
1.524 AU = 2.280 $\times$ 10^8 km

Maximum light time
21.0 min

Minimum light time
4.4 min

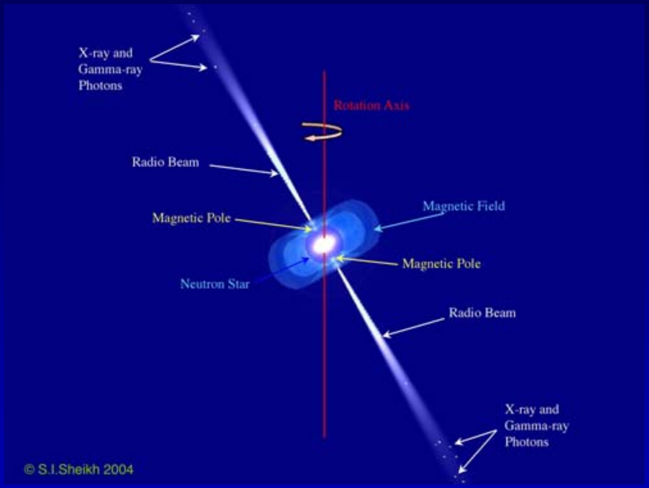
Pulsar timing

Crab Nebula

Remnant of supernova observed on Earth in 1054

Optical spectrum

X-ray spectrum

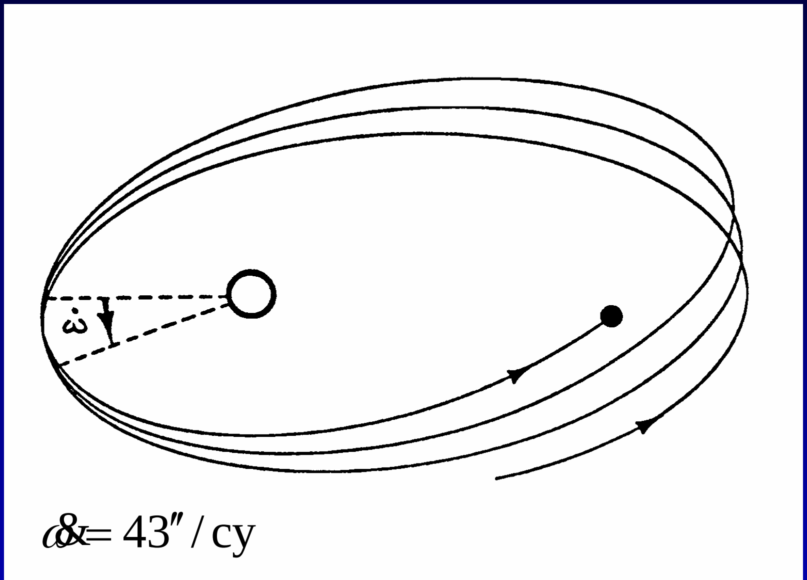


Pulsar at center

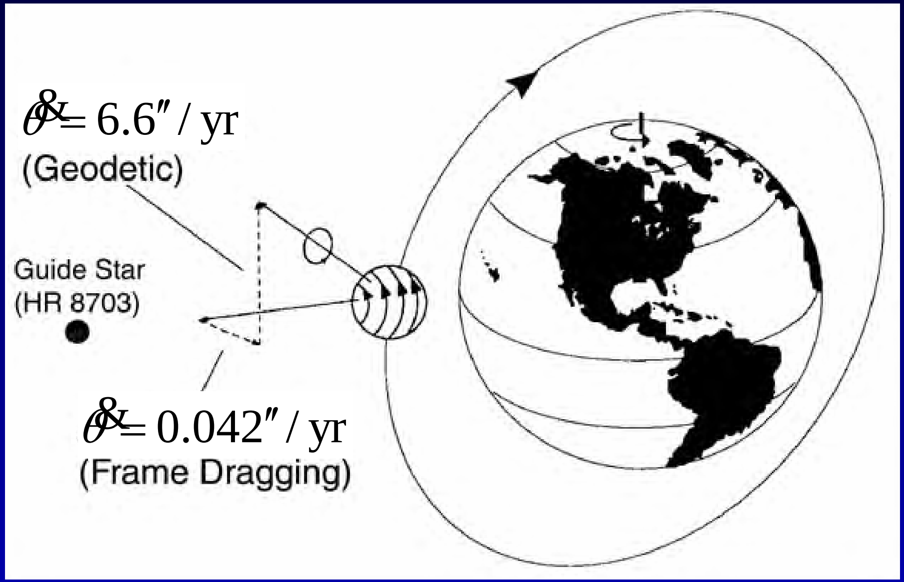
At the center of the bright nebula is a rapidly rotating neutron star (pulsar) that emits electromagnetic pulses over a wide bandwidth with a period of 33 ms.

X-ray pulsars can be used as precise time references. Relativistic transformations from the pulsar inertial frame to the solar system barycentric frame and then to the geoid frame will be required.

Precessional effects



Precession of perihelion of Mercury



Gravity Probe B

Equation of motion to post-Newtonian order

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3}\mathbf{r} - \frac{1}{c^2}\frac{GM}{r^3}\left[\left(4\frac{GM}{r} - \dot{\mathbf{r}}\cdot\dot{\mathbf{r}}\right)\mathbf{r} + 4(\mathbf{r}\cdot\dot{\mathbf{r}})\dot{\mathbf{r}}\right] + 2\frac{1}{c^2}\frac{GM}{r^3}\left[\frac{3}{r^2}(\mathbf{r}\cdot\mathbf{J})\mathbf{r} - \mathbf{J}\right]\times\dot{\mathbf{r}} + 3\frac{1}{c^2}\frac{GM}{r^3}(\mathbf{r}\times\dot{\mathbf{r}})\times\dot{\mathbf{r}}$$

Newtonian acceleration

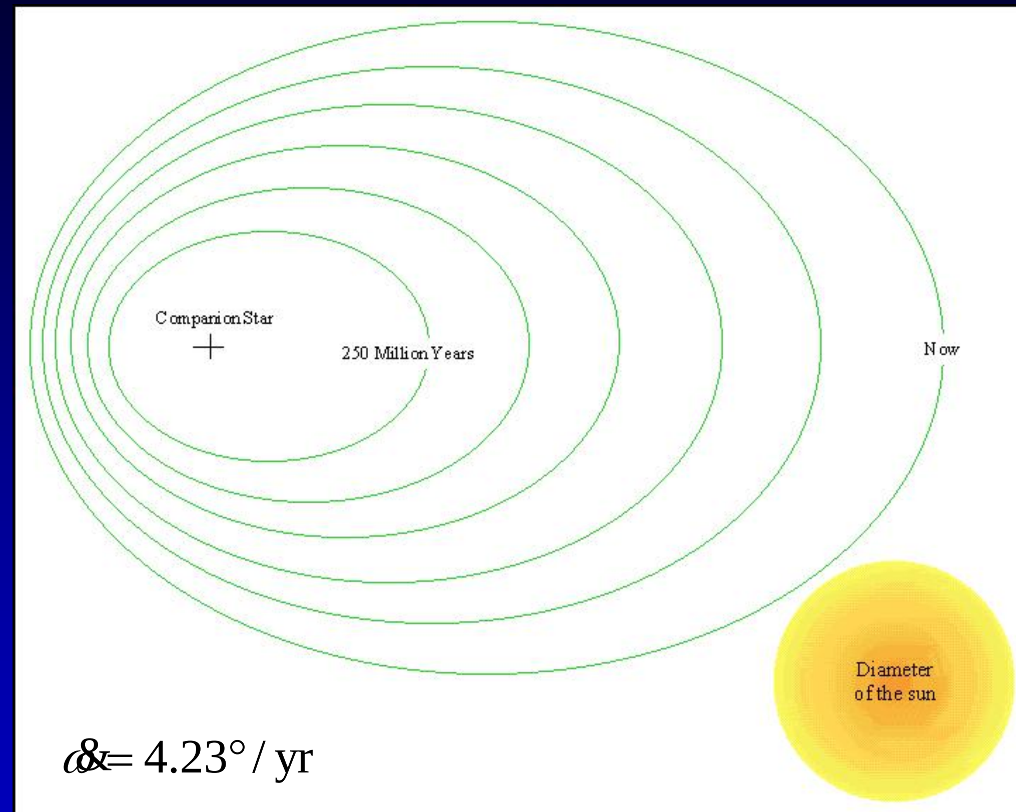
Precesssion of periapsis

Lens-Thirring precession (frame dragging)

Geodetic (de Sitter) precession

Gravitational waves

Joseph Weber at the University of Maryland



Binary pulsar PSR 1913+16

Joseph Weber founded the field of gravitational wave astronomy with his invention of the bar detector.

In 1993, the Nobel Prize in physics was awarded to Russell Hulse and Joseph Taylor of Princeton University for their 1974 discovery of the binary pulsar PSR 1913+16 and their analysis of its emission of gravitational waves, corresponding to a rate of loss of energy in agreement with general relativity.

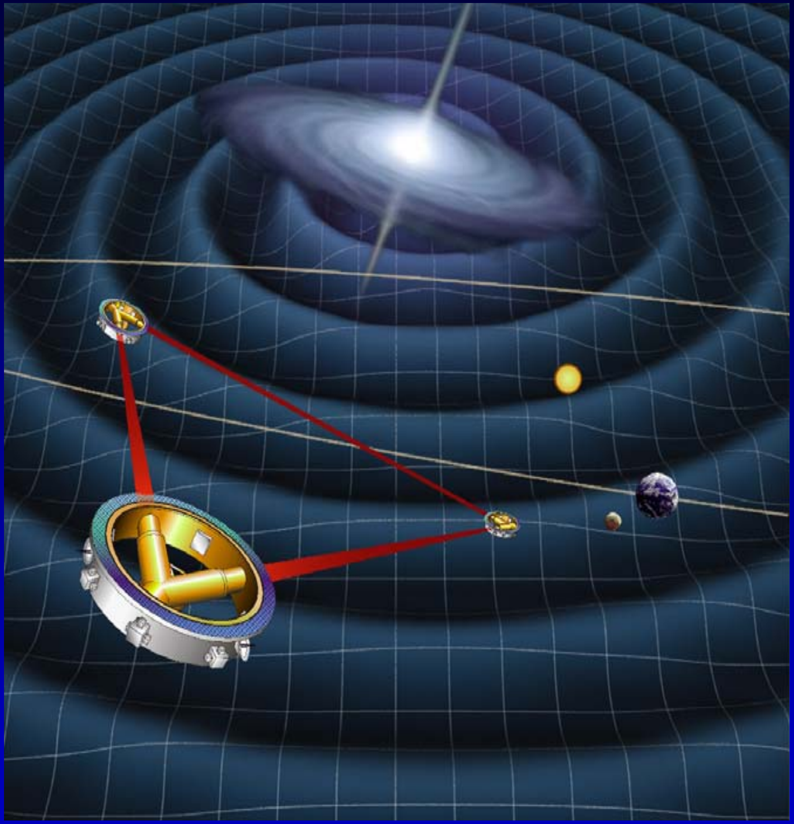
Laser interferometer GW antennas

Livingston, Louisiana

Hanford, Washington

Laser Interferometer Gravitational-Wave Observatory (LIGO)

Interferometer arms are 4 km long. System is designed to observe gravitational waves in the bandwidth of 10 Hz to 5000 Hz



Laser Interferometer Space Antenna (LISA)

Three heliocentric spacecraft separated by 5,000,000 km form an interferometer to observe gravitational waves in the bandwidth of 0.001 Hz to 1 Hz

Summary

- As clock technology and theory have progressed, time scales and methods of time measurement have evolved to achieve greater uniformity and self-consistency
- Astronomical measures of time and been replaced by atomic measures of time
- High precision time measurement and dissemination has required considerations of the principles of the special and general theories of relativity
- The GPS has provided a model for relativistic time measurement
- Similar considerations will be required in the development of new systems, such as Galileo, and interoperability with these systems
- The GPS provides a model for navigation and the dissemination of time throughout the solar system

Today the general theory of relativity is not simply a subject of theoretical scientific speculation, but rather it has entered the realm of practical engineering necessity.